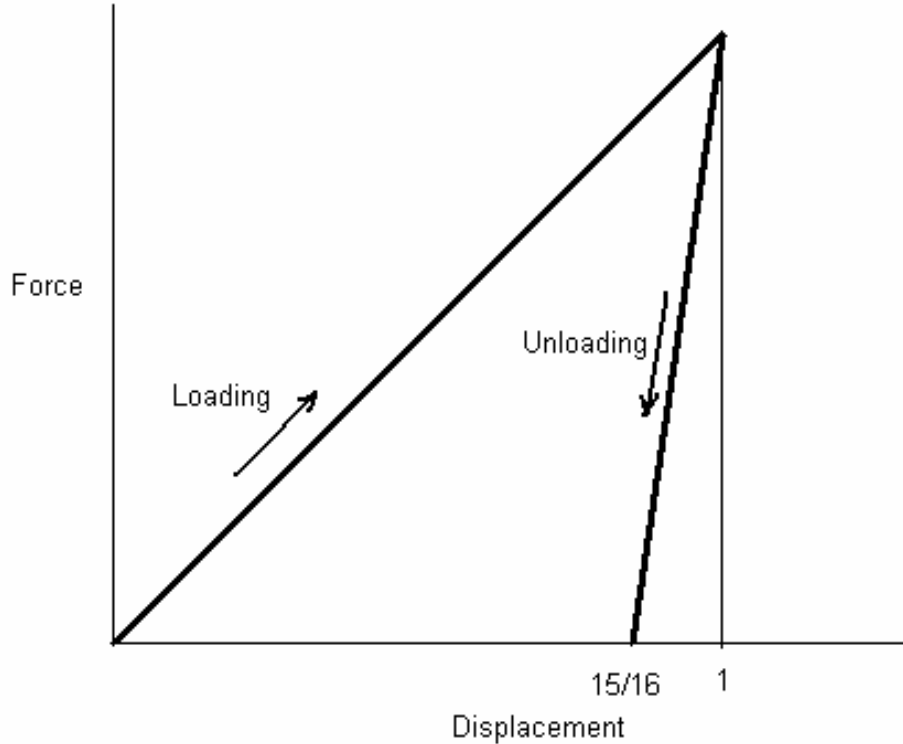


I propose the following simple helmet model to study the way increased impact velocity might the time durations of the helmet response at or over 200 G. Assume that the helmet behaves like a linear spring during the loading phase and that it unloads linearly as well but with 16 times the spring constant in the loading phase.



Since the area under the unloading curve is only a sixteenth the area under the loading curve, the rebound velocity at the end of the impact is 25% of the impact velocity. The impact acceleration trace looks like a quarter sine wave going from 0 to 90 degrees butted up against another quarter sine from 90 degrees back down to 180 degrees but four times faster.

Given the peak G's and the impact velocity, it is possible to calculate the helmet properties and the durations of the sine wave components as well as the total time the impact acceleration remains at or above 200G.

The durations of the quarter sine components are:

$$t_L = \frac{\pi}{2} \frac{V_0}{P_G * 9.81}$$

$$t_U = \frac{\pi}{2} \frac{V_0/4}{P_G * 9.81} = 0.25 * t_L$$

The total duration at or over 200G would be:

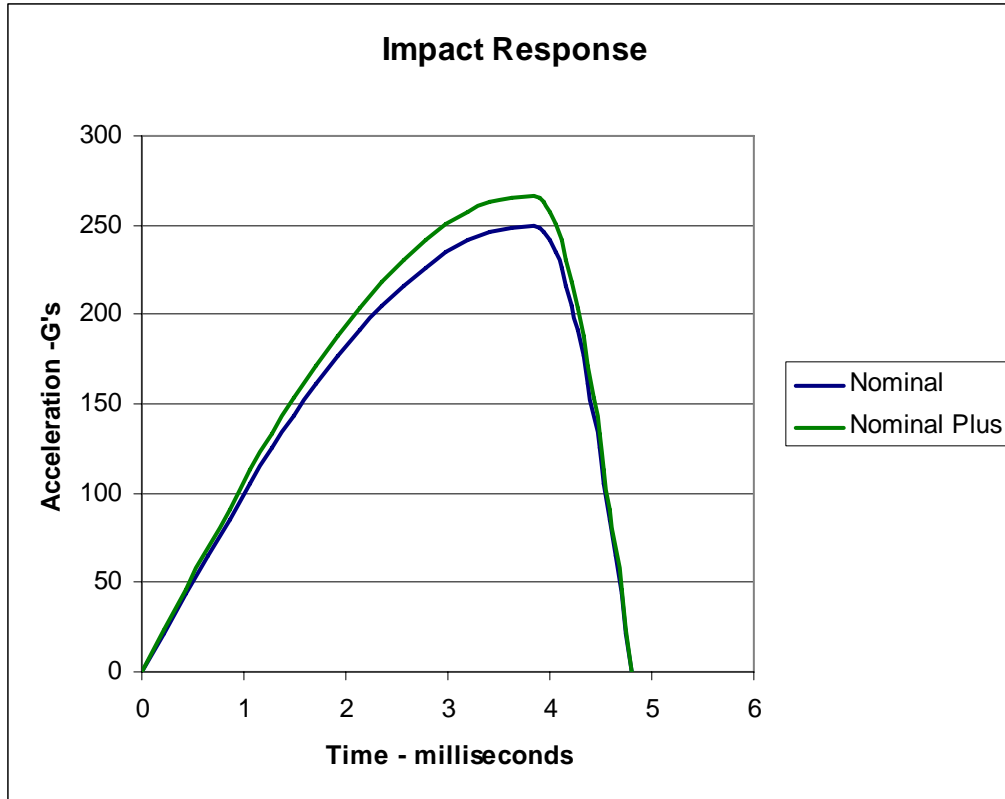
$$t_{@200G} = 1.25 * (1 - 2 * \arcsin(200 / P_G) / \pi) * t_L$$

Where t_L is the loading time in seconds, t_U is the unloading time in seconds, $t_{@200G}$ is the total duration at or over 200G in seconds, V_0 is the impact velocity in meters per second and P_G is the peak acceleration in G's. The arcsin function should be calculated in terms of radians.

The same helmet impacted at higher velocities will obtain proportionately higher peak accelerations but the durations of the sine wave components will remain the same. The mathematics is the same as for pendulums and plucked strings. However, the durations at and over 200G will change. Effectively, the velocity increase pushes the pointy ends of the sine waves farther up through the 200 G threshold so, even though the total pulse duration is unchanged, the duration at 200 G increases. The culprit is the arcsin function. Everything in the equation remains the same except P_G which increases proportionately with impact velocity. As P_G increases, the arcsin calculation decreases causing the calculated value of the time duration to go up.

Velocity	6 m/sec			6.4 m/sec			% change
	Peak G	Pulse Time	@200G	Peak G	Pulse Time	@200G	@200G
	G's	msecs	msecs	G's	msecs	msecs	
Helmet#1	250	4.8036585	1.967893	266.6667	4.8036585	2.210196	12.31%
Helmet#2	240	5.0038109	1.865716	256	5.0038109	2.14746	15.10%
Helmet#3	230	5.2213679	1.716777	245.3333	5.2213679	2.053224	19.60%
Helmet#4	220	5.4587028	1.493257	234.6667	5.4587028	1.912988	28.11%
Helmet#5	210	5.7186411	1.12802	224	5.7186411	1.700692	50.77%
Helmet#6	201	5.9746996	0.379571	214.4	5.9746996	1.401978	269.36%

The table shows the results of calculations for six different hypothetical helmets. All the helmets behave as in the first figure but helmet#1 is sufficiently stiff to obtain a 250 G peak for a 6 meter per second impact while helmet#6 is so soft that it obtains only 201 G in a similar impact. The first set of gray columns shows the helmet responses to 6 meter per second impacts and the second set shows the same information for a 6.4 meter per second impact. The total pulse times remain the same even though the second impact is at a 6.7% higher velocity. But the peak accelerations are greater and the durations at or above 200 G are substantially greater. The 6.7% increase in velocity balloons to increases from 12% to much, much more for time durations at or above 200 G. Helmets #1, 2 and 3 go from comfortably meeting requirements at the impact velocity specified in FMVSS 218 to failing at the maximum velocity proposed for compliance testing.



The figure above shows the expected impact responses for helmet#1 in the table for a nominal 6.0 meter per second impact and for a “nominal plus”, 6.4 meter per second impact which would be permitted under the proposal. As described above, the traces start from the left and ascend as quarter sine waves to their peak values; then they drop in similar quarter sine wave shapes but four times more quickly back to zero G’s. Comparing the two traces shows a considerable expansion in the time duration at and over 200 G even though the total pulse times are the same. In this example, the nominal trace meets the 2 millisecond time duration but the “nominal plus” trace fails.

Although these results are hypothetical, they are reasonable approximations of reality. Other reasonable models of helmet behavior will yield similar changes in time duration criteria.